Shaping Mathematics Education that Allow the Learners to Pursue Further Learning

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Abstract

In this paper, a process of learning and development related to the idea of noticing is proposed to allow learners to have a better progress in their learning process to achieve the objectives they have already set prior to the learning. There should be an intentional aspect during the learning, especially when objects to be learned are considered as difficult by many learners, such as mathematics. There will be some challenging non routine problems the learners will be facing and certainly similar challenges will also be met in their career upon accomplishing their formal education or in their further learning. In order for the people to develop to their optimum capacity they must do their best in adjusting to the changes in their environment, for example in the technology communication and information. Teaching mathematics which starts from early ages to at least high school should not only promote characters and beauty of mathematics but character education and skills as well to trigger a good habit of mind in the part of the learners. Mathematics teachers must notice and be sensitive to didactical and pedagogical situation to support the learners.

Keywords: Shaping, Mathematics education, noticing, non routine skill

Introduction

Mathematics is considered as an important science in humans’ life, such that, since children started their formal education until they reached high school, mathematics is still a compulsory subject. At school, mathematics favors more time periods as to compare to other subjects. If one decided to proceed to a higher level of education than high school, then she/he might as well encounter mathematics or other subjects for example economy, technology, astronomy, etc. that for certain will require she/he to use her/his mathematical thinking and skills in doing mathematics. Learning mathematics, according to Schoenfeld (in Henningsen & Stein, 1997) is an active and generative process conducted users of mathematics, which provides opportunity for them to develop their mathematical thinking.

As such, learners who studies mathematics should experience learning processes and activities which are designed in such a way that in the end they must take certain final exams to ensure that they succeed. Consequently, in formal education it is a requirement for the learners who should take final tests/exams to find out about their ability which opens new opportunities for them to be successful in their life.
Demands and challenges in general will always be faced by people, therefore it is required through education that the learners have a high quality skills in order for them to survive and be productive, particularly in global competition nowadays. Therefore the learners should take exams and tests and they are expected to be successful in the exams before they could proceed to a higher level of education or to apply for a new job. These tests indicate a form of competition in order to recruit qualified persons. At this stage, actually, there is a challenge such that in order for those persons to pass such tests then they should be well prepared.

In relation to the competitiveness, it is often seen that there is always a trend of how the learners were under the teachers’ guidance or how the learners work seriously as well as working independently or being a self directed learners. Since mathematics is one of the subjects included in the final exams, it is commonly understood that teachers as well as their learners focus more generally on the substance of mathematics. However, it should be taken into account that not only mathematics that the learners will need in the future, but the impact of the characteristics of mathematics on the learners’ affective aspects, such as attitude, motivation, values, independency etc should be considered as an important issue. In other words, teaching mathematics at schools will also involve the characteristics of mathematics and therefore such characteristics may have certain impacts on the learners’ character.

**Characteristics of Mathematics**

In the process of doing mathematics, people need to be sensitive to the empirical phenomena that they have experienced and in making sense of the information they got. In this process, people will create concepts and extend it, and through abstraction they will come to generalization followed by its application on similar problems. Generally, mathematics always concerns about generalization and also considered as a science which allows opportunities for people who study it to think creatively, critically and logically.

Several characteristics of mathematics, among others are as follows: Applicability and effectiveness, abstraction and generality, simplicity, logical derivation, axiomatic arrangement, precision, correctness. (Ebrahim, 2010).

Having good mathematics knowledge and skills will allow effectiveness in its application in solving problems encountered in learning or in life. Skills in utilizing mathematics concepts and strategies help the learners to apply mathematics in solving problems effectively. Abstraction is a form of generalization of particular and interesting issues in mathematics. In generalization, the essence of a subject is identified and its essence is then systematically organized nicely. Generalization is an effort to organize many things in detail and it will form a manageable and a well organized framework. The result of an abstraction is then expressed in a simple representation to show its simplicity, for example a mathematics formula is a form of generalization. Often times the learners are asked to express an algebraic form in a simple representation, for example \( n! \) (\( n \) factorial) is used to represent \( 1.2.3...n \).

Logic and axiomatic characteristic was initiated firstly in the works of Euclid in geometry, when in fact prior to Euclid’s works, mathematics was an empirical science. Mathematical language is very efficient and serves as a powerful tool in expressing mathematical ideas or concepts, exploring and in reconstructing the results as well as in communicating mathematics. Mathematical language is certainly not ambiguous but it is practical and shows its clarity as in definitions or in theorems.
People who studies, teaches or uses mathematics, of course may have certain characteristics or habits as a result of doing mathematics and applying mathematical procedures or rules in thinking mathematically. For example in producing a proof about a statement, a learner must use certain relevant theorems or must correctly choose and apply certain formula to come to a solution. He or she must comply with the rules. Very often the problem that must be solved is difficult and challenging that a learner must struggle and might get stuck and spends a lot of time before he/she could solve the problem. In the meantime, the learner must be consistent and persistent so as not to give up easily. The learners learn how to be patient and curious in solving mathematics problems. It could be understood that the “learning mathematics’ environment” somehow may have certain impacts on the habits or on the character of mathematics learners or mathematics teachers.

Character

The real character or originality of character of an individual or a group of people is represented in the actions or how things are expressed in words or their spontaneity. A number of values, among other: honesty, courage, risk taking, respect on differences and appreciation could be seen in how people react on a certain event or how people solve a problem. Character is represented by a specific reflection of something specific inside a person or a group of people. In Kebijakan Nasional (2010:7), character is perceived as specific good values (knowing good things, want to do good things, live a good life, care about and own a good impact on the environment) integrated in one self and appear in one behavior (Winataputra, 2010). Therefore, character is considered as an integration of cognitive ability (mind), feeling, physics and works of an individual or a group of people. These specific things are represented in the form of the adopted values, moral and potential to act upon facing a critical and challenging situation.

The role or function of character is considered as essential and strategic. In connection to the forming of the character, as cited by Husaini (2011) according to Indonesian Minister of National Education, that character education should be started at the early age, that whenever a good character has been built, it could not be changed easily. Building a good character is something that every human being wants. For that purpose, every individual should learn to develop and after the individual had developed, he/she will be able to learn more (Mason, 2002). Therefore, the process of building good characters will always have opportunities to take place as long as the individuals are in the process of learning.

Educational process at school to build and to develop good habits and characters is not just an occasional activity, added only when it is considered necessary by teachers, but it is more likely to be a different approach in teaching and learning process provided deliberately to learning environment. To promote and to shape good habit and character to the learners it is imperative that the teacher should do and model such character in the classroom. It should be integrated in the instruction that the teacher deliberately has designed and planned to do it in class. It is expected to have certain impacts on the part of the learners and on the part of the teachers as well.

Regarding the instruction that the teacher already designed to be implemented in the classroom, the teacher should have certain intentions relating to promoting good habits and characters to learners. During the process of learning, learning activities that the teacher already designed, learners may work collaboratively in group or work individually. The learners will have to interact actively with their peers or with the teacher. Whenever certain aspect appears related to character, the teacher should be always ready to respond, or showing a good model of such character. In this case, the teacher should notice and be aware and sensitive to what is taking place in the learners’ interaction among themselves. The teacher
may respond right away, or make notes about what had happened and plan to respond later related to the habit or character that the teacher wants it to be learned by the learners. The teacher should not avoid responding to the learners otherwise she/he will lost the good momentum to model such character to the learners.

In the design that the teacher already prepared, for example, deliberately the teacher will encourage the learners to argue or to exchange their strategies in solving interesting and challenging or difficult mathematics problems. Such problems might potentially create certain cognitive conflicts or the learners could not come up with a solution. In this situation, every learner must work seriously, independent, or work in groups, patiently and do not give up easily. The learners as well must try to respect different ideas, or communicate and appreciate good solutions offered by other learners. The presence of the teacher is not only to guide the learners in finding the solution, but also to respond to learners’ attitude so that the teacher may model certain character or to encourage the learners to perform such character. Encouraging the learners to work independently and to be a self directed learner is important, after working together in groups. Because even the learners must learn to work together in group they must learn how to work alone or independent to be a self develop learner, because somehow a student also must learn at home by herself/himself. It is imperative that the learners are accustomed to working together and working individually and to having guidance from the teacher related to character building. Therefore the teacher should the advantage of such a situation designed on purpose or not to promote character education for the learners.

Therefore, one of the roles of the teacher is to create a didactical situation (Brousseau, 1997). According to didactical situation theory, the teacher should create a situation as a starting point to learning process. Although such situation does not immediately create a learning process but with certain conditioning, for example, through scaffolding, such learning process might as well take place. If the learning process takes place, then there will be new situation as a consequence of the learner actions or responses on the previous situation. The new situation could be unique or more than one situation depends on the potential of the learners and the learning activities that the teacher designed (Suryadi, 2010).

Since in mathematics classroom there will be always problems encountered by the learners in solving those problems, therefore a number of values could be identified and detected within an individual learner or group of learners, among other: honesty, courage, risk taking, respect on differences and appreciation of how one reacts on a certain event or a problem. The teacher could make use of certain mathematics challenging problem to learn about learners’ character and then followed by promoting a good character to the learners. For this purpose, open-ended problem is considered as a good media that might trigger the emergence of a good character. In open-ended problems the learners might come up with different strategies or different true solution. And this situation will certainly prepare a good opportunity for the teacher to perform or model good character to the learner as well as the learner will learn about such good character.

The presence of character or integrating character building in the process of learning mathematics at school actually is not a new issue. Many educators have already came to a conclusion that practically every intended activity stated in the curriculum have provided opportunities that enable character building to take place. As such the mathematics teachers must make sure that learning activities in mathematics classroom is potentially relevant to implement character education (Cowley, 1936). This is in line with didactical situation that the teacher should design to create learning process (Brousseau, 1997). Schumacker (in Rice, 2011) argued that it is important for the teachers to encourage the learners to determine the objectives that they want to achieve as well as to do their best in learning and to have positive attitudes. For these purposes the learners should be assisted or be involved in planning and
designing relevant and appropriate learning programs to assure that the teachers will be responsible to achieve what they have already programmed for the learners. This could also be considered as the teachers’ commitment to help the learners through their positive actions and their positive thinking about the learners.

Combining the theory of didactical situation and Schumacker’s opinion, the mathematics teachers must as well involve the learners in determining not only objectives related to mathematics achievement but also related to character building of the learners.

Mathematics, although is difficult for many learners, has some special characteristics or aspects that are potentially useful to help the learners to have excellent qualification. Certain competencies that are expected to be owned by the learners are among others: think critically, creatively, ability in mathematical reasoning and problem solving. In solving non routine problems, the learners are often got stuck so they must give their best effort to be able to solve the problem. Therefore, it is required that the teachers should design learning activities and choose relevant problems, approaches and strategies with relevant methods in order to trigger the emerging of certain mathematics competencies. This would mean that the teachers must be sensitive and be aware of the strategic function of mathematics and mathematics education. However, besides cognitive competencies, the learners must also have other skills such as ability to work in groups as in cooperative learning, work independently, comply with the law, and be responsible, appreciative and respective on difference solutions or strategies especially in discussing whether a solution is right. The learners will also learn to make good judgment and come to conclusion before making a right decision.

Different competencies and skills should be introduced and developed in learning mathematics through solving complex and non routine problems. The learners should experience it in essence that they are accustomed to facing and solving problems and they might need to apply their skills and experience whenever they face difficult and complex problems yet challenging in their life and in their career in the future. The difficulties and challenges are not supposed to be avoided but should be considered as something which may create opportunity and expectation for the learners to be successful in solving it. At the same time when the learners see this opportunity, they will do their best and work hard and become consistent and persistent. In the learning activities the learners could collaborate or may solve such complex problems cooperatively or also solve the problem independently.

Regarding the challenges that will always be faced, the learners should be guided and be encouraged in risk taking. To solve mathematics problems concerning problem solving situation, the learners should be encouraged to apply different strategies relevant to the problems as well as different level of solutions. The learners should be ready to get stuck, however they need not to give up right away. They must be ready to persevere in facing difficult problems. If the learner is always discourage in taking risk or in making mistakes then they will never learn how to solve difficult problems whereas such difficult situation will always be part of their life in the future. The teachers should not let the learners be discouraged in facing difficult problems. Instead the teachers should be facilitators who assure the learners that the teachers will facilitate them in solving the problems. There must be assurance that whenever the learners make mistake they will not be punished immediately, instead they must be assisted to find the solution. This would mean that the teachers are required to anticipate the possibility of the mistakes made by the learners and should not let them be discouraged with the difficulties in mathematics that they often encounter. The teachers will have to be ready to facilitate the learners with hints or clues in supporting the learners that might be used by the learners to produce right solution.
What the teachers demonstrate in supporting the learners indicates how the teachers care for the benefit of the learners. Caring attitude from the teachers is required as compared to letting the learners to make mistakes and gradually fail in their learning activity. This caring attitude really shows how the teachers are concerned with the learners’ success or are concerned with their future. To implement the caring attitude the teachers are required to notice intentionally, be sensitive and be ready to model how they care as often as possible so that the learners realize that caring is a good character they must have. The learners will then apply such character (caring) to help others who would want to be successful in their life.

Mathematical problems that are designed and posed in a way to create challenges to the learners should be open-ended. In this kind of problem the learners are expected to solve the problem using more than one strategy. This situation will allow the learners to use level of knowledge they have to solve the problem. The learners will realize that in the different solutions and strategies they could still find the truth in their solutions. Therefore, it is surely a good opportunity for the learners to adopt new ideas and at the same time learn to respect differences in the solutions and strategies. These varieties in strategies and solutions may be considered as something privilege offered by mathematics. Through open-ended problems the teachers already created a rich learning environment such that the learners they could learn from their classmates.

Actually one of the essences of working on open-ended problems is that the learners will learn how to respect the difference and to appreciate the truth in these differences as well. In other words truth in mathematics could appear in many ways.

**Wonderful Mathematics**

Difficulty in mathematics is certainly a fact to many learners and the learners will always encounter it when they study mathematics. Then, what should the teacher do to help the learners to understand and interest in to doing mathematics? The teacher should have certain objectives for the learners in and certain teaching approaches and methods relevant to it. These objectives are not only about cognitive aspects, but also about affective aspects. In connection to further learning in the future, then the learners must have rich experiences in doing mathematics, and how they enjoy the beauty and the wonder of mathematics. Then the learners as well as the teachers deserve to have nice environment and supports, facilities provided by schools. These supports and facilities will enable the teacher to design better learning process and activities to facilitate the learners to achieve the goals that the teachers and the learners set together. It is expected that the effect of good environment and the relevant designs will enhance the learner to enjoy studying even new difficult teaching materials.

The nice learning environment will also motivate the learners to be self regulated learners that they will learn many new things to meet certain needs or requirement. Nevertheless, the learners might need assistance from their peers or from the teacher, because they surely encounter difficult problems, especially designed by the teacher to trigger interactions among the learners or between the teacher and the learners.

Whenever the learners could sense how wonderful mathematics is, then curiosity could emerge within their mind and force them to be eager to know further what makes all this wonderful mathematics happened. The emerging of the curiosity will somehow create opportunities and trigger the learners to explore and to uncover the secret behind the wonder of mathematics. They will show their best effort and become persistent and will not give up before they uncover the secret. Upon finding the secret or the
reasons behind the wonder of mathematics, the learners will begin to develop their mathematical disposition, namely: perseverance, interest and curiosity, appreciation of the role and the strength of mathematics, flexible, reflect on what has been achieved and self confidence.

The wonder and the beauty of mathematics are detected whenever at the beginning of a situation a learner did not expect that some amazing structure will appear and suddenly he/she realizes certain patterns or representations and some mathematics concepts begin to play its role in clarifying something that initially did not seem to have any relation at all with the situation. The wonder and the beauty of mathematics that is deliberately emerged are supposed to inspire and invite the learners to learn mathematics. Creating a situation that is followed by forming a good perception about mathematics may serve as an opening to integrate character education. The teachers then must take advantage of this situation and intentionally use this momentum to integrate character building related to the values beyond the beauty and the wonder of mathematics. The teachers then could model a certain character and values that they want the learner to have it. It means that the teachers should notice a right moment and serve as facilitator and motivators and be ready to act before losing such a good opportunity to integrate the character education.

A mathematics teacher should be aware, sensitive, caring and supporting in the learning and teaching process. These sensitivity and awareness are needed in the term of instructional quality improvement, that learners should improve their knowledge. Related to the notion of noticing, actually noticing are needed as a something that related to the future, but it should be done in this time and use every single thing that notice in this time as a preparation (Mason, 2002).

The awareness of the teacher in indentifying the condition in learning and teaching process and also in analyzing it immediately is required. It will help the teacher to determine and decide relevant action or response that should be taken such that the instructional process is not stop and still in its lines. Sometimes during the instructional process in the class, there are some unpredictable situations that can serve as a chance that can be very useful to be developed and become a new sub-process that can give many benefits for learners. If the teacher had good capability in doing noticing, such kind of conditions can be very important and also very useful to build a strong motivation of the learners. But on the opposite, if the teacher are not aware with such kind of condition, then the opportunity that come will be gone without any advantages.

Since the teachers must intentionally create a didactical situation to trigger right momentum to initiate character building and values in mathematics classroom, they must make a design and plan an appropriate lesson. After deciding what mathematics topic/problems will be presented in class, the teachers might list the kind of characters (perseverance, appreciation, comply with the rule, appreciation, respect, hard work) they want the learners to learn about and also contextual problems (mathematics topic) relevant to the character and value. The topic is presented and the chosen problem is then solved and emphasized on the beauty and the wonder of mathematics integrated in the problem. Certainly as already planned, the learners should be able to respond (reflect) on the topic pertaining to the strategy, structure and how systematic the problem was solved. Once the learners responded, the teachers should model the character they intended the learners to have. The learners then will try to keep it in their mind or even imitate such character. And finally the teachers and the learners will try to perform such character in classroom at another time or outside the classroom. Following is a simple diagram of how character education might be integrated in mathematics classroom.
Following is a simple example of how from a seemed simple situation and question the beauty and the wonder of mathematics may emerge,

**Example:**
*Observe the following situation and its problem* (Sabandar, 2011)

A delegation consists of 3 individuals will be selected from a group of people.

How many delegations can be formed, if a delegation is selected from a group of 3, 4, 5, 6, 7, 8, 9 … or n people?

**Solution**

1. To select from 3 people.
   
   Let the group has A, B, C, then there will be only 1 (one) possible choice….. namely,
   
   **A B C**
   
   *(Only 1 choice)*

2. To select from 4 people. Let the group has A, B, C, D …then there will be 4 possible choices… namely,
   
   **A B C**
   **A C D**
   **A B D**
   **B C D**
   
   *4 choices*
3. If selected from 5 people A, B, C, D, E

Then there will be 10 possible choices and arranged in the following representation:

\[
\begin{align*}
\text{A B C} & \quad \text{A C D} & \quad \text{A D E} \\
\text{A B D} & \quad \text{A C E} \\
\text{A B E} & \\
\text{B C D} & \quad \text{B D E} \\
\text{B C E} & \quad \text{C D E} \\
\end{align*}
\]

(10 choices)

4. If selected from 6 people A, B, C, D, E, F

Then there will be 20 possible choices and are arranged in the following representation:

\[
\begin{align*}
\text{A B C} & \quad \text{A C D} & \quad \text{A D E} & \quad \text{A E F} \\
\text{A B D} & \quad \text{A C E} & \quad \text{A D F} \\
\text{A B E} & \quad \text{A C F} \\
\text{A B F} & \quad \text{B C D} & \quad \text{B D E} & \quad \text{B E F} \\
\text{B C E} & \quad \text{B D F} \\
\text{B C F} & \quad \text{C D E} & \quad \text{C E F} \\
\text{C D F} \\
\text{D E F} & \quad \text{(20 choices)} \\
\end{align*}
\]

If the structures in the first, second, third and forth representation are displayed using numbers and be put in a table, then we could observe it in the following Table 1.
Table 1: Structures Displayed in Numbers, Selecting 3 Elements from 3,4,5,6,7…Elements

<table>
<thead>
<tr>
<th>Select 3 from</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structures</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Number of choice</td>
<td>1</td>
<td>4</td>
<td>10</td>
<td>20</td>
<td>35</td>
<td>…</td>
</tr>
</tbody>
</table>

We could display these structures into the following representations, which in turn may provide a simple way (after observing the new representations) to determine the number of the next choices. This could be done without repeating the process to form the structure such as at the beginning of the process.

In this example the teacher deliberately chooses a situation and problems (questions) that challenge the learner to do mathematics. At first the learners might try to solve the problem informally, and with the facilitating that come from the teacher they will come up with a nice structure that they never expected it to appear. With this nice structure, they could see a pattern that will allow them to make a prediction and then they test their answer, for example by using the structure the learners may find the number of the possible delegation selected from a group of 20 people. The learners could extend this structure and may enable them to see the nice pattern, which could be used to answer similar question without referring to a formula.

This situation could be followed by another mathematical situation arises from learners curiosity which based on their mathematical knowledge to come to generalization stage through a formal approach facilitated by the teacher. The teacher and the learners may want to utilize some mathematical concepts and strategy, such as representations in the form of functions and tables in generating a formula for choosing three people from a group of \( n \). In the end the learner will have a formula to answer similar question or similar situation.

In order to attract the learners curiosity and motivation to learn mathematics so that there will be opportunity for the teacher to model and to promote good character to the learners, it should be taken into account the way a topic or a problem is presented by the teacher. How the teacher facilitates the learners and shows models how to think mathematically is a necessity. The teacher should have known the learners strengths and weaknesses, and be ready to notice and care to respond to learners actions related to mathematical concepts or to character that the teacher should promote and model. Once the good
characters are familiar to the learners and they could perform it gradually, then his character may have impacts on their habit in learning or habits of mind in learning.

Conclusion

This presentation actually shows a model of how to inspire the learners to make hypothesis and followed by examining possible solutions to assure that the solution is correct. It is expected that the beautiful and wonderful aspects of mathematics when presented in an interesting and attractive way may trigger the curiosity and then force them to want to learn mathematics. Such situation in learning mathematics where the learners are motivated and inspired by the performance of the teacher in teaching and learning mathematics would provide a good momentum for the mathematics teachers to integrate character education. This means that not only a certain priority (for example: attention, time, care, support) goes to learning mathematics but also this momentum might be used optimally to develop good characters of the learners. This will also mean that not only mathematics knowledge be emphasized in learning mathematics but also it is expected that mathematics characteristics will also have impacts in forming good characters of the learners, especially when the teachers as well as the learners are ready to notice a right momentum in modeling and in forming the intended good characters. At the same time those characters identified in the process of learning should be well communicated by the teachers to the learners continually.

Related to the forming and developing good characters, it is really expected that the teachers should prepare some beautiful and wonderful mathematics, as well as good characters to be exposed to the learners. In this case, mathematics teachers should be trained and be prepared to facilitate and expose the beauty and the wonder of mathematics in the topics that the learners are about to learn. Moreover, such characters should be modeled by the teachers showing the beauty and the wonder of mathematics and followed by comments about that beauty and the wonder of mathematics which will encourage the learners to study further in their life.

References


