

WHICH IS BETTER: HOMOSKEDASTIS OR HETEROSKEDASTIS VOLATILITY MODEL FOR OPTION PRICING? (TEST OF BLACK-SCHOLES OPTION MODEL AND GARCH OPTION MODEL AT THE INDONESIAN STOCK EXCHANGE)

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ABSTRACT

The purpose of this research is to compare the accuracy of Black-Scholes Option Model and GARCH option models for Stock option utilizing data from Astra, BCA, Indofood and Telkom at the Indonesian Stock Exchange. The intraday stock return of Astra, BCA, Indofood and Telkom exhibits an overwhelming presence of volatility clustering; suggesting that GARCH model has an effect which best corresponds with the actual price. The best model is constructed using ARIMA model and the best lag in GARCH model is extracted. The finding from this research show that by comparing the average percentage mean squared errors of the GARCH Option Model and the Black-Scholes Option Model, the former was found more accurate than the latter. GARCH Model relatively improves average percentage mean squared errors of Black-Scholes Model ; one month option shows a twenty eight point ten percent improvement, two month option shows twenty three point thirty percent and three month option shows twenty percent.

Keywords: ARIMA, Black-Scholes Option Pricing Model, GARCH Option Model, Indonesian Stock Exchange.

INTRODUCTION

A key determinant of option value is the volatility of the underlying asset. Amongst the five variables in the Black Scholes Option Pricing Model (1973) only the volatility is not observable. In the Black Scholes Option Pricing Model , one often assumes that the volatility is constant over time.

Volatility is a central part of most asset pricing models. Volatility also affects the sensitivity of option value with respect to the asset price, option's time to maturity, interest rate and variance itself. Thus, in principle, an accurate spesification of variance could have significant bearnings on the value of options, and the calculation of option value sensitivity.

Among finance and economics application , the most widely used family of models for modelling volatility is the *AutoRegresive Conditionally Heteroskedastic* (ARCH) introduced by Engle (1982) where the autoregressive property in principle mean that old events leave wave behind a certain time after actual time of the action.

The term "conditional heteroskedasticity" mean that the variance (*conditional on the available information*) varies and depends on old value of the process. One can resemble this with the process having a short – term memory and that the behavior of the process is influenced by this memory.

The simplest possible spesification for the variance, or volatility would be :

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 \dots\dots\dots(1)$$

A practical problem encountered in fitting ARCH (*p*) models to financial return data was that in order to obtain a good fitting model, the order *p* needed to be fairly large. This potentially leads to the problem of negative variance and non stationerity.

Bollerslev (1986) introduced the class of *Generelized AutoRegresive Conditionally Heteroskedastic*. Though the name is quite mouthful, the form of GARCH(*p,q*) models of order (*p,q*) is relatively simple ;

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2 \dots\dots\dots(2)$$

Bollerslev showed that the GARCH process is covariance stationary if $\sum_{i=1}^p \alpha_i + \sum_{i=1}^q \beta_i < 1$ it implies that the effect of the past squared innovation on the current conditional variance decays exponentially with the lag length.

The purpose of this research is to determine whether GARCH option model using the best lag of GARCH is more accurate in forecasting pricing stock option premia than Black-Scholes model at the Indonesian Stock Exchange. Average percentage mean squared error is used to find the best model .

RESEARCH REVIEW ON GARCH OPTION PRICING MODEL

Heston dan Nandi(2000) used GARCH(1.1) model to compare the accuracy with Black-Scholes Model and using average mean squared error to find the best model . Using intraday S&P 500 index option traded on the Chicago Board Option Exchange (CBOE). The intraday data set was sampled every Wednesday starting from 1992 to 1994. Heston dan Nandi found that GARCH (1.1) model on the S&P 500 index options data shows substantial pricing improvement over the Black-Scholes Model.

Fofana dan Brorsen (2001), used GARCH (1.1) t-distribution Option model with implied volatility to compare the accuracy with Black-Scholes Option Model and using average mean squared error to find the best model. Using daily data Chicago wheat option premia from Wall Street Journal during July 1987 to July 1993, Fofana and Brorsen found The GARCH OPM with implied volatility outperforms the Black OPM with implied volatility at options close to maturity (6 to 15 days). For options ranging from 16 to 20 days to maturity, the Black OPM with implied volatility is as good as the GARCH OPM with implied volatility. For options ranging from 21 to 50 days to maturity, the Black OPM with implied volatility outperforms the GARCH OPM with implied volatility. Thus, neither model dominates the other

Harikumar, Boyrie dan Pak (2004) , evaluated the accuracy of Black-Scholes Option model dan GARCH option pricing model using Currency Exchange and Currency Call Option Data. Harikumar, Boyrie dan Pak used GARCH – in Mean to predict the volatility of British Pound, Japanese Yen and Swiss Franc during 5 January 1987 to 29 December 1995.

Data was collected from Wharton Research Data Base Service Foreign Currency, currency call option from Philadelphia Stock Exchange (PHLX) currency option data base and using average percentage mean squared error to find the best model. Harikumar, Boyrie dan Pak found that in British Pound, Swiss Franc and Japanese Yen, Black-Scholes Model more accurate than GARCH(1.1)-in Mean Model, and GARCH(3.3) in Mean Model. And also found that GARCH(1.1)-in Mean Model more accurate than GARCH(3.3)-in Mean Model.

DATA AND METHODE OF ANALYSIS

Data

To estimate variance, we used secondary intraday dataset provided from the Indonesian Stock Exchange from January to March 2005. Captured every 30 minutes, 885 data tick were taken from Astra International, Bank of Central Asia , Indofood Sukses Makmur and Telkom Indonesia.

To validate the model, we used secondary intraday data of stock option contract from April to June 2005 and closing prices of stock from May to August 2005 were to determine the performance result.

Building Model

The formation of the best GARCH model is done through several processes, which include:

a. Root Unit Testing

Root unit determination is done by DF test uses Critical Value from David MacKinnon. Based upon Dickey Fuller Test, the outcome is derived at level ASTRA Stock data non-stationary due to the DF value statistically = -2,3303 > critical values (5%) = -2,8646. From testing on first difference with a result of stationary data, due to DF value = -33,9902 < critical values (5%) = -2,8646.

On stock level of Bank BCA non stationary data demonstrates DF value statistically = -1.0562 > critical values (5%) = -2,8646 and from testing on first difference, is recovered a result of stationary data due to DF value-statistically = -30,0303 < critical values (5%) = -2, 8646.

On stock level of Indofood non stationary data demonstrates DF value statistically = $-1.2997 >$ critical values (5%) = $-2,8646$ and from testing on first difference, is recovered a result of stationary data due to DF value-statistically = $-37,0154 <$ critical values (5%) = $-2, 8646$.

On stock level of Telkom non stationary data demonstrates DF value statistically = $-1.8106 >$ critical values (5%) = $-2,8646$ and from testing on first difference, is recovered a result of stationary data due to DF value-statistically = $-37,6641 <$ critical values (5%) = $-2, 8646$.

b. Selecting the best ARIMA lag to form GARCH model

Based on unit root testing, it is determined that the best ARIMA model for Astra stock is as follows:

Table 1: ARIMA Model on ASTRA Stock

Astra International	AIC	SIC	Prob
AR (1)	11.06657	11.07740	0.00
AR (16)	11.08969	11.10067	0.00
AR (1), (16)	11.07266	11.08913	0.00

Sources : Research finding

As shown on Table 1 above, it is then determined that the best model used to estimate ARCH/GARCH is model AR(1), because it has the least value of AIC and SIC, compared to other estimating models.

Selected for BCA stock ARIMA model is as follows:

Table 2: ARIMA Model on BCA Stock

BCA	AIC	SIC	Prob
AR (15)	8.944684	8.955656	0.00
AR (32)	8.955465	8.966609	0.00
AR (15, 32)	8.949100	8.965817	0.00

Sources : Research finding

As shown on Table 2 above, it is then determined that the best model used to estimate ARCH/GARCH is model AR(15), because it has the least value of AIC and SIC, compared to other estimating models.

Selected for Indofood stock ARIMA model is as follows:

Table 3: ARIMA Model on Indofood Stock

Indofood	AIC	SIC	Prob
AR (1)	8.131039	8.141873	0.00
AR (11)	8.169799	8.180731	0.00
AR (33)	8.146543	8.157698	0.00
AR (36)	8.151639	8.162825	0.00
AR (1) AR (11)	8.123380	8.139778	0.00
AR (1) AR (33)	8.109107	8.125839	0.00
AR (1) AR (36)	8.107645	8.124424	0.00
AR (11) AR (33)	8.130311	8.147044	0.00
AR (11) AR (36)	8.136309	8.153088	0.00
AR (33) AR (36)	8.129067	8.145846	0.00

AR (1)			0.00
AR (11)	8.091630	8.113940	0.00
AR (33)			0.00
AR (1)			0.00
AR (11)	8.091079	8.113451	0.00
AR (36)			0.00
AR (11)			0.00
AR (33)	8.113666	8.136038	0.00
AR (36)			0.00
AR (1)			0.00
AR (33)	8.091079	8.113451	0.00
AR (36)			0.00
AR (1)			0.00
AR (11)			0.00
AR (33)	8.073036	8.101001	0.00
AR (36)			0.00

Sources : Research finding

As shown on Table 3 above, it is then determined that the best model used to estimate ARCH/GARCH is model AR(1,11,33,36)), because it has the least value of AIC and SIC, compared to other estimating models.

Selected for Telkom stock ARIMA model is as follows:

Table 4: ARIMA Model on Telkom Stock

Telkom	AIC	SIC	Prob
AR (1)	9.280213	9.291747	0.00
AR (13)	9.332206	9.343158	0.00
AR (1,13)	9.275242	9.291670	0.00

Sources : Research finding

As shown on Table 4 above, it is then determined that the best model used to estimate ARCH/GARCH is model AR(1,13), because it has the least value of AIC and SIC, compared to other estimating models.

c. Selecting the best lag of GARCH :

Based on ARIMA estimating model on ASTRA stock, the formation of ARCH/GARCH model is as follows:

Table 5: GARCH Model on ASTRA Stock

MODEL	$p=1$	$p=2$	$p=3$	$q=1$	$q=2$	$q=3$	AIC	SIC
ARCH (1)	0.144	-	-	-	-	-	-7.5079	-7.4863
ARCH (2)	0.147	(0.015)	-	-	-	-	-7.5082	-7.4811
ARCH (3)	0.154	(0.024)	0.032	-	-	-	-7.5072	-7.4747
GARCH (1,1)	0.130	-	-	(0.191)	-	-	-7.5087	-7.4816
GARCH (1,2)	0.145	-	-	(0.176)	0.015	-	-7.5068	-7.4743
GARCH (1,3)	0.100	-	-	(0.275)	0.117	0.151	-7.5043	-7.4663
GARCH (2,1)	0.149	0.021	-	(0.307)	-	-	-7.5073	-7.4748
GARCH (2,2)	0.143	(0.104)	-	0.551	0.192	-	-7.5083	-7.4704
GARCH (2,3)	0.126	(0.097)	-	0.565	0.213	0.033	-7.5068	-7.4635
GARCH (3,1)	0.165	(0.183)	0.005	1.006	-	-	-7.5746	-7.5367
GARCH (3,2)	0.136	(0.098)	0.003	0.527	0.171	-	-7.5046	-7.4613
GARCH (3,3)	0.127	(0.073)	0.002	0.370	0.131	0.043	-7.5010	-7.4522

Sources : Research finding

As shown on Table 5 above, based on AIC and SIC comparison, ASTRA stock GARCH model (3.1) is selected as the best model because it has the least of AIC and SIC value.

Based on ARIMA estimating model on BCA stock, the formation of ARCH/GARCH model is as follows:

Table 6: GARCH Model on BCA Stock

MODEL	$p=1$	$p=2$	$p=3$	$q=1$	$q=2$	$q=3$	AIC	SIC
ARCH (1)	0.199						-7.2989	-7.2770
ARCH (2)	0.199	0.037					-7.3025	-7.2751
ARCH (3)	0.202	0.036	0.009				-7.3005	-7.2676
GARCH (1,1)	0.198			0.267			-7.3040	-7.2765
GARCH (1,2)	0.204			0.221	0.061		-7.3024	-7.2695
GARCH (1,3)	0.098			(0.127)	0.199	0.742	-7.3183	-7.2799
GARCH (2,1)	0.180	(0.156)		0.948			-7.3182	-7.2853
GARCH (2,2)	0.184	(0.164)		1.063	(0.108)		-7.3171	-7.2787
GARCH (2,3)	0.226	0.102		(0.381)	0.064	0.267	-7.3351	-7.2912
GARCH (3,1)	0.184	(0.141)	(0.020)	0.950			-7.3171	-7.2787
GARCH (3,2)	0.189	0.030	(0.177)	0.176	0.746		-7.3342	-7.2903
GARCH (3,3)	0.178	0.030	(0.159)	(0.037)	0.740	0.189	-7.3455	-7.2962

Sources : Research finding

As shown on Table 6 above, based on AIC and SIC comparison, BCA stock GARCH model (3.3) is selected as the best model because it has the least of AIC and SIC value.

Based on ARIMA estimating model on Indofood stock, the formation of ARCH/GARCH model is as follows:

Table 7: GARCH Model on Indofood Stock

MODEL	$p=1$	$p=2$	$p=3$	$q=1$	$q=2$	$q=3$	AIC	SIC
ARCH (1)	0.161						-6.0321	-5.9930
ARCH (2)	0.108	(0.011)					-6.0210	-5.9762
ARCH (3)	0.080	(0.009)	0.704				-6.2927	-6.2423
GARCH (1,1)	0.051			0.954			-6.3776	-6.3228
GARCH (1,2)	0.075			0.385	0.548		-6.3788	-6.3285
GARCH (1,3)	0.106			(0.039)	0.361	0.588	-6.3792	-6.3233
GARCH (2,1)	0.098	(0.050)		0.949			-6.3798	-6.3294
GARCH (2,2)	0.049	0.076		(0.095)	0.818		-6.3231	-6.2672
GARCH (2,3)	0.077	(0.057)		1.409	(0.413)	(0.018)	-6.3691	-6.3076
GARCH (3,1)	0.048	0.015	0.646	0.100			-6.3115	-6.2556
GARCH (3,2)	0.092	0.084	(0.059)	(0.045)	0.860		-6.3660	-6.3045
GARCH (3,3)	0.090	(0.075)	(0.008)	1.549	(0.408)	(0.147)	-6.3694	-6.3023

Sources : Research finding

As shown on Table 7 above, based on AIC and SIC comparison, Indofood stock GARCH model (2.1) is selected as the best model because it has the least of AIC and SIC value.

Based on ARIMA estimating model on Telkom stock, the formation of ARCH/GARCH model is as follows:

Table 8: GARCH Model on Telkom Stock

MODEL	p=1	p=2	p=3	q=1	q=2	q=3	AIC	SIC
ARCH (1)	0.558						-7.8147	-7.7873
ARCH (2)	0.199	-0.020					-7.7703	-7.7374
ARCH (3)	0.223	-0.018	-0.002				-7.7772	-7.7389
GARCH (1,1)	0.557			-0.013			-7.8129	-7.7801
GARCH (1,2)	0.386			-0.044	-0.006		-7.8033	-7.7649
GARCH (1,3)	0.387			-0.029	-0.015	-0.009	-7.8042	-7.7604
GARCH (2,1)	0.247	-0.131		0.483			-7.8239	-7.7956
GARCH (2,2)	0.234	-0.116		0.438	0.005		-7.7788	-7.7350
GARCH (2,3)	0.220	-0.103		0.402	0.008	0.001	-7.7695	-7.7202
GARCH (3,1)	0.293	-0.140	0.001	0.433			-7.7935	-7.7497
GARCH (3,2)	0.241	-0.107	-0.006	0.399	0.017		-7.7790	-7.7297
GARCH (3,3)	0.216	-0.087	-0.012	0.371	0.015	0.005	-7.7554	-7.7007

Sources : Research finding

As shown on Table 8 above, based on AIC and SIC comparison, Telkom stock GARCH model (2,1) is selected as the best model because it has the least of AIC and SIC value.

Method of Analysis

We used average percentage mean squared error to obtain the best model.

$$AMSE = \frac{1}{N} \sum_{t=1}^N \left(\frac{AP_t - SP_t}{AP_t} \right)^2 \dots\dots\dots(7)$$

Where :

- APt = Actual Premia of Option
- SPt = Simulated Premia of Option
- N = Number of simulation

RESEARCH FINDING

The accuracy test for one month option

The average percentage mean squared error in one month option on ASTRA stock option for Black –Scholes Option Model is 4.51 %, BCA is 3.62 %, Indofood is 8.94 % and Telkom is 4.68 %. Meanwhile for GARCH (3.1) on Astra stock option average percentage mean squared error is 3.92 %, GARCH (3.3) on BCA stock option average percentage mean squared error is on BCA is 8.53 %, GARCH (2.1) on Indofood stock option average percentage mean squared error is 3.66 % and GARCH (2.1) on Telkom stock option average percentage mean squared error is 4.23 %.

From the results shown above, it shows that by comparing the average percentage mean squared errors of the GARCH Option Model and the Black-Scholes Option Model , the former was found more accurate than the latter. GARCH Option Model relatively improves average percentage mean squared errors of Black-Scholes Option Model ; one month option shows a twenty eight point ten percent improvement.

The accuracy test for two month option

The average percentage mean squared error in two month option on ASTRA stock option for Black –Scholes Option Model is 5.44 %, BCA is 8.68 %, Indofood is 13.12 % and Telkom is 11.71 %. Meanwhile for GARCH (3.1) on Astra stock option average percentage mean squared error is 4.72 %, GARCH (3.3) on BCA stock option average percentage mean squared error is on BCA is 8.11 %, GARCH (2.1) on Indofood stock option average percentage mean squared error is 9.58 % and GARCH (2.1) on Telkom stock option average percentage mean squared error is 8.74 %.

From the results shown above, it shows that by comparing the average percentage mean squared errors of the GARCH Option Model and the Black-Scholes Option Model, the former was found more accurate than the latter. GARCH Option Model relatively improves average percentage mean squared errors of Black-Scholes Option Model; for two month option shows twenty three point thirty percent.

The accuracy test for Three month option

The average percentage mean squared error in two month option on ASTRA stock option for Black-Scholes Option Model is 2.71 %, BCA is 6.84 %, Indofood is 11.74 % and Telkom is 4.74 %. Meanwhile for GARCH (3.1) on Astra stock option average percentage mean squared error is 2.55 %, GARCH (3.3) on BCA stock option average percentage mean squared error is on BCA is 6.74 %, GARCH (2.1) on Indofood stock option average percentage mean squared error is 10.23 % and GARCH (2.1) on Telkom stock option average percentage mean squared error is 3.01 %.

From the results shown above, it shows that by comparing the average percentage mean squared errors of the GARCH Option Model and the Black-Scholes Option Model, the former was found more accurate than the latter. GARCH Option Model relatively improves average percentage mean squared errors of Black-Scholes Option Model; for three month option shows twenty percent.

CONCLUSION

This research confirms that the best model to calculate option premia is done with GARCH model deriving from ARIMA model and the best lag in GARCH model. The finding from this research show that by comparing the average percentage mean squared errors of the GARCH Option Model and the Black-Scholes Option Model, the former was found more accurate than the latter. GARCH Model relatively improves average percentage mean squared errors of Black-Scholes Model; one month option shows a twenty eight point ten percent improvement, two month option shows twenty three point thirty percent and three month option shows twenty percent.

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